The Influence of the Stress Path on the Characteristic Stress State

Ibsen, Lars Bo; Praastrup, U.; Jakobsen, Kim Parsberg

Publication date:
1999

Document Version
Early version, also known as pre-print

AAU Geotechnical Engineering Papers

ISSN 1398-6465 R 9909

The influence of the stress path on the characteristic stress state

K.P. Jakobsen, U. Praastrup, L.B. Ibsen

September 1999

Soil Mechanics Paper No 32

GEOTECHNICAL ENGINEERING GROUP
AALBORG UNIVERSITY DENMARK

Jakobsen, K.P., Praastrup, U., Ibsen, L.B. (1999). The influence of the stress path on the characteristic stress state.

AAU Geotechnical Engineering Papers, ISSN 1398-6465 R9909.

Soil Mechanics Paper No 32

The paper has been published in *Proc.2nd Int. Symp. Pre-failure Deformation Characteristics of Geomaterials*, IS Torino 99, Sept. 26-29.1999, pp. 659-666.

The **AAU** **G**eotechnical **E**ngineering **P**apers - AGEP - are issued for early dissemination and book keeping of research results from the Geotechnical Engineering Group at Aalborg University (Department of Civil Engineering). Moreover, the papers accommodate proliferation and documentation of field and laboratory test series not directly suited for publication in journals or proceedings.

The papers are numbered ISSN 1398-6465 R<two digit year code><two digit consecutive number>. For internal purposes the papers are, further, submitted with coloured covers in the following series:

Series	Colour
Laboratory testing papers	sand
Field testing papers	grey
Manuals & guides	red
Soil Mechanics papers	blue
Foundation Engineering papers	green
Engineering Geology papers	yellow
Environmental Engineering papers	brown

In general the AGEP papers are submitted to journals, conferences or scientific meetings and hence, whenever possible, reference should be given to the final publication (journal, proceeding etc.) and not to the AGEPpaper.

2nd Int. Symp. Pre-failure Deformation Characteristics of Geomaterials, IS Torino 99, 28-30 Sep. 1999

The influence of the stress path on the characteristic stress state

K.P. Jakobsen
Aalborg University, Aalborg, Denmark

U. Praastrup
Aalborg University, Aalborg, Denmark

L.B. Ibsen
Aalborg University, Aalborg, Denmark

ABSTRACT: Volume change is the core phenomenon in understanding the behaviour of frictional materials, such as sands. In conventional drained triaxial compression tests the transition from contraction to dilatation occurs along a well-defined line in triaxial stress space, the characteristic line. However, little information exists about the effect of stress path on the characteristic line. A test program has been carried out to scrutinise this effect. The tests were designed to emanate from the hydrostatic axis and pass through a characteristic stress state obtained from a conventional triaxial test. In case of stress path independency all tests should start to dilate at the same state of stress. The results show an indubitable dependency of the stress path and that the characteristic friction angle is constant for a given type of stress path. It is further examined how these findings comply with the theory of plasticity.

1 INTRODUCTION

The core to understanding the stress-strain behaviour of frictional materials, such as sands, lies in understanding the effect of volume change and in particular the various factors that influences the volume change. Experimental observations show that factors influencing the volume change are relative density, stress level, stress path, drainage conditions and properties of the test material.

Element tests, such as triaxial tests, are performed in order to provide the understanding of the stress-strain behaviour of frictional materials and to support the development of constitutive models. Only tests performed under drained conditions are considered in this paper, but the findings may apply to undrained tests as well.

Volume changes can be either compressive or expansive in nature. The analysis of several triaxial tests reveals that expansive or dilative volume changes are most pronounced for dense specimens sheared at low confining pressures. Conventional triaxial compression tests performed on dense specimens at low pressures demonstrate that the specimens initially contract and subsequently dilate. This state where the transition from contraction to dilation occurs is entitled the characteristic state (Luong 1982).

This paper focuses on the characteristic state and in particular the stress equivalent entitled the characteristic stress state. In the view of developing more accurate constitutive models it is important to consider how the characteristic state is captured. It is important because this state significantly influences the progress of the volumetric strain. A constitutive model that is not able to capture this state cannot predict the stress-strain behaviour of frictional materials correctly.

One of the components in an elasto-plastic constitutive model is the plastic potential surface, which is closely related to the characteristic stress state. The state of stress on the plastic potential surface farthest off the hydrostatic axis controls the plastic transition from contraction to dilation. This corresponds to the characteristic stress state except for an elastic contribution.

Due to this relation, it is important to be familiar with the factors that influence the location of the characteristic stress state. The factor or issue investigated here is whether the stress path in triaxial tests has any effect on the location of the characteristic stress state. The effect of stress path on the state is studied experimentally by carrying out two series of triaxial tests.

2 BASIC IDEA

The basic idea behind this paper is to investigate whether the characteristic stress state is stress path independent as claimed by Luong (1982). The first series of tests is especially designed to verify or

The tests are performed in a renewed version of the Danish triaxial apparatus. The working principles of the apparatus were developed in the late sixties by Jacobsen (1970). The newly developed version uses similar working principles, but the data acquisition has been completely modified and enhanced together with automatic load control. Measurements of axial load, cell pressure, pore pressure, volume change and axial displacement are automatically and electronically collected and transmitted to a computer for storage and processing. Any stress path within the failure envelope can be followed automatically and very precisely.

3.1 Specimen preparation and testing procedure

In order to facilitate the present study a combination of relative density and stress level that enhances the size of the contraction zone and still features dilation is chosen. For that reason the tests are performed on medium dense specimens sheared at moderate confining pressures.

The specimens are prepared in a cylindrical split mould by air pluviation with an initial void ratio of 0.672, tolerating a deviation of ± 0.001.

Since all measurements merely represent average quantities, it must be required that the stress and strain state inside the specimen are homogeneous throughout the test. This requirement is met by using lubricated end plates and preparing the specimen with a height and diameter of approximately 70 mm (see for example Ibsen 1993).

In the triaxial apparatus the specimen is saturated using a combination of the vacuum and water percolation method (Jakobsen & Praastrup 1998). Subsequently the specimens are isotropically consolidated at a maximum loading rate of 5 kPa per minute and afterwards sheared at a constant axial strain rate of 3.0% per hour. The specimens are all sheared into the softening regime and terminated at an axial strain of approximately 10%.

3.2 Analysis of triaxial tests

The results are presented in terms of Cauchy or true stresses and non-linear logarithmic strains. This approach leads to a consistent set of formulas as it is based on an exact representation of the displacement field and uses a finite strain measure for both axial and volumetric strains. It also has the advantage that the stress and strain measures are work conjugated (Praastrup et al. 1998).

4 STRESS PATH DEPENDENCY

A series of tests are performed in accordance with the principles outlined in Section 2 in order to investigate the stress path dependency of the characteristic stress state. The test program shown in Figure 1 is carried out at two different stress levels. Consequently, two conventional triaxial compression tests are initially performed as their characteristic stress states determine the initial isotropic stress state for the remaining tests. The results of the two conventional tests are listed in Table 2.

Table 2. Characteristic stress states for conventional triaxial compression tests.

Test no.	σ'_3 [kPa]	p'_{cl} [kPa]	q_{cl} [kPa]	φ'_{cl} [°]
971005	320.0	539.7	659.3	30.5
971006	160.0	272.9	338.6	30.9

There is according to Figure 1 only one type of the tests that follows a non-monotonous stress path. In this test the mean effective stress and subsequently the deviator stress is held constant. The soil can according to the elasto-plastic theory exhibit either elastic or elasto-plastic behaviour. The non-monetonous test and in the test where the axial effective stress is held constant the specimens are sheared into the elastic region. These two tests make it possible to examine some elastic effects on the characteristic stress state (see Section 4.2). In the other three types of tests the specimens are plastically sheared throughout the test.

The test results are shown in Figure 2. The different stress paths are shown in $p'-q$ diagrams whereas the measured strains are shown in combined $\varepsilon_1 - q - \varepsilon_v$ diagrams. The characteristic stress states and failure states are marked by open and closed circles, respectively.

4.1 Evaluation of characteristic stress states

The stress-strain curves in Figure 2 reveal that the characteristic stress state can only be determined in the tests with increasing and constant mean effective stress (4, 5, 6, 15, 22 and 23). The figure also reveals that the location of the characteristic stress state differs significantly. The characteristic angles for the specimens sheared under constant mean effective stress (tests 4 and 15) are lower than the angles observed in the two conventional tests. The deviation in the angle between test 5 and 15 is 15.1%, whereas the deviation between test 4 and 6 is 13.6%. The characteristic angles for specimens sheared with a slope of 2 in the $p'-q$ plane are higher that the angles observed in the two conventional tests. The deviations are found to 4.9% and 5.8%. The deviations between the highest and the lowest angle is as high as 26.3%. The magnitude of these deviations leads to the conclusion that the characteristic stress state is indeed dependent on the stress path. On the other hand, it is observed that the characteristic

Table 3. Results of triaxial tests.

Test no.	σ'_3 [kPa]	p'_{cl} [kPa]	q_{cl} [kPa]	φ'_{cl} [°]
971004	176.0	271.7	287.1	26.7
971015	354.8	538.3	550.5	25.9
971022	170.6	304.0	400.1	32.7
971023	337.1	590.1	759.1	32.0

Surprisingly, it is impossible to determine the characteristic stress states for the tests with constant axial effective stress and constant deviator stress (3, 17 and 33). The specimens sheared at constant axial effective stress tests do not contract at all, whereas both contraction and dilation occur in the test with constant deviator stress (Figure 2b). However, the transition from contraction to dilation is caused entirely by the sudden change of stress path. Such a transition is not in accordance with the definition of the characteristic state.

4.2 Numerical analysis of tests 17 and 33

The stress-strain behaviour of soils can be modelled by use of elasto-plastic constitutive models. In order to investigate the behaviour in tests 17 and 33 the single hardening model has been employed. The single hardening model is an advanced elasto-plastic constitutive model for frictional material such as soils, concrete and rock (Kim & Lade 1988; Lade & Kim 1988a,b Lade & Nelson 1987). A hypoelastic model describes the elastic behaviour, whereas the framework for plastic behaviour consists of a failure criterion, a yield criterion and a non-associated flow rule.

The failure criterion bounds a domain of possible stress states and simply determines the maximum load that a soil element can withstand. As the plastic deformations, within the domain of possible stress states, follow a non-associated flow rule the yield criterion and the plastic potential surface are described by different functions. The yield criterion controls whether plastic deformations occur. The plastic potential surface controls the direction of the plastic strain increments. At the top point of the plastic potential surface, in the $p' - q$ plane, the plastic strain increment vector is perpendicular to the hydrostatic axis corresponding to a zero increment in plastic volumetric strain (Wood 1994). The plastic strain increment vector will, depending on the current state of stress, point in the inward or outward direction of the hydrostatic axis, indicating plastic dilation or contraction, respectively.

Consequently, the characteristic state and the top point of the plastic potential surface are closely related. The relation between the two is due to the fact that the plastic volumetric strain increment is zero on the top of the plastic potential surface, whereas the characteristic state occurs where the "total" volumetric strain increment is zero. The trace of the top point of the plastic potential surface in the $p' - q$ plane should, therefore, except for an elastic contribution, reflect the trace of the characteristic stress states.

The single hardening model has been employed to draw the curves in Figures 3 and 4. The figures show two sets of yield and plastic potential surfaces labelled "Y" and "P", respectively. Y_A-curves correspond to the yield surfaces prior to elastic shearing and the P_A-curves to the plastic potential surfaces which are activated when the soil starts to yield again. Y_B and P_B refer to the yield and plastic potential surfaces that passes through the characteristic stress state found in the conventional triaxial tests (5 and 6). The characteristic states for these tests are marked with open circles, whereas the failure points for the investigated tests are marked with closed circles.

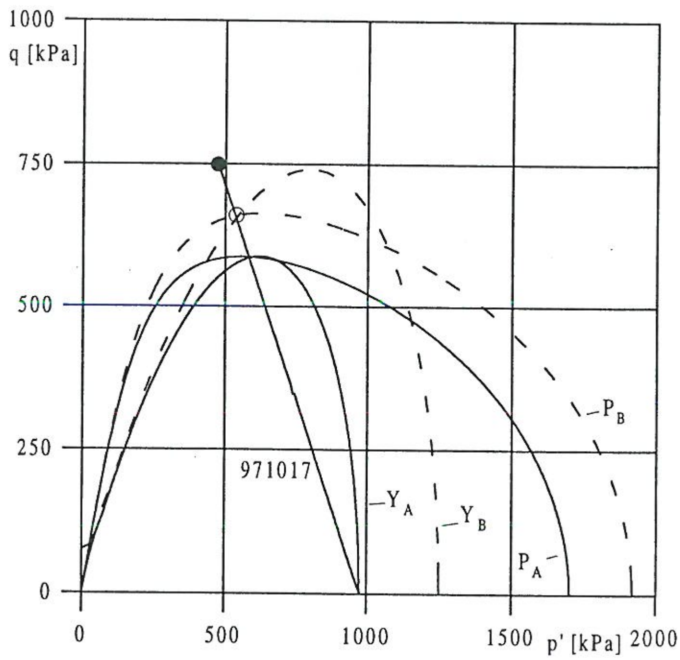

Figure 3. Yield and plastic potential surfaces. Test 971017.

A comparison of Figures 2b and 3 reveals that the yield criterion of the single hardening model captures the first yield point very well. Both figures show that the soil behaves purely elastic until a deviator stress of approximately 580 kPa. Subsequently, the soil starts and continues to yield throughout the test. The most interesting portion of the stress path is between the point of first yield and the characteristic stress state observed for the conventional triaxial test. The shape of the two plastic potential surfaces indicates that the top of the plastic potential surface is passed somewhere between P_A and P_B along the stress path shown in the figure.

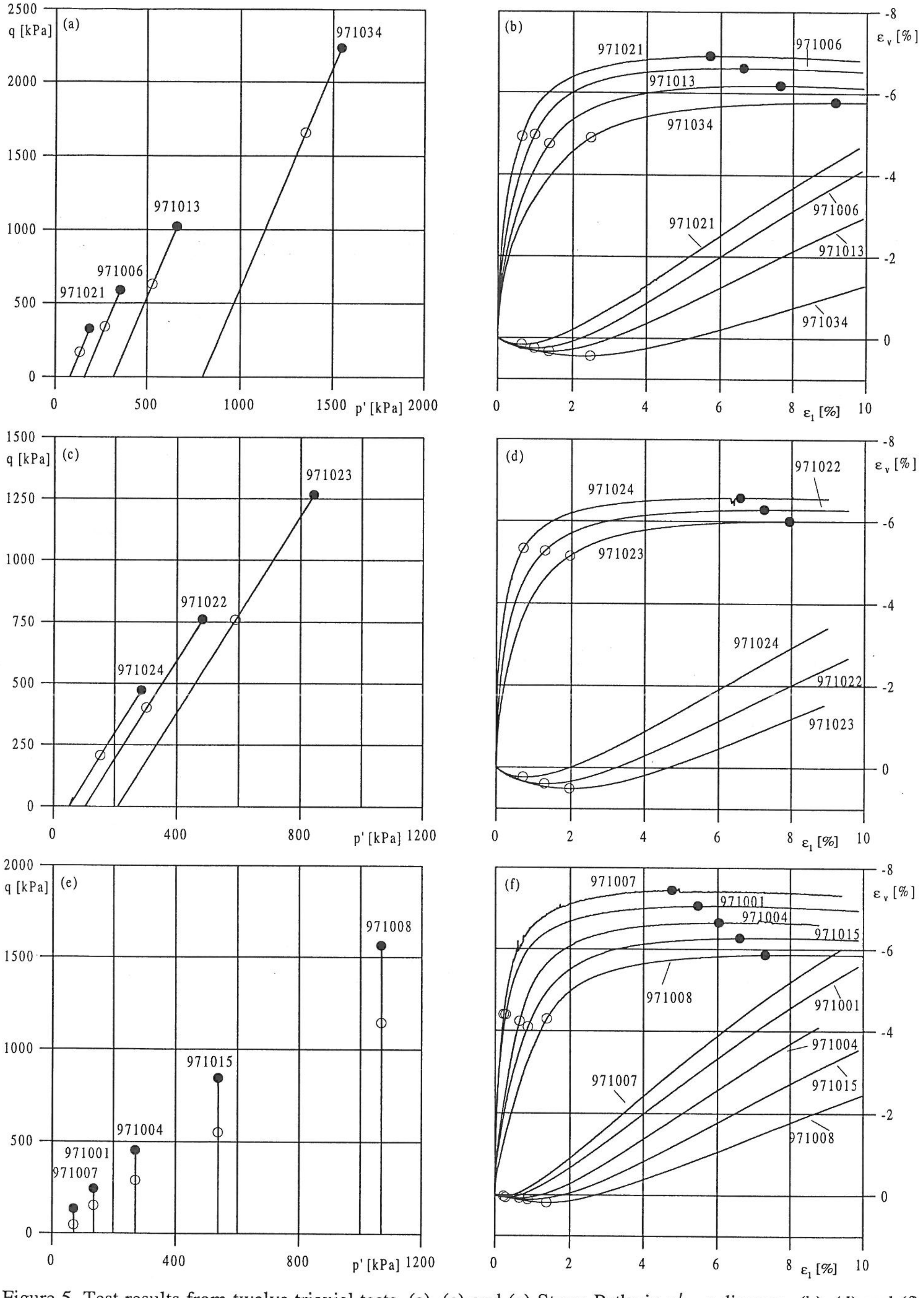

Figure 5. Test results from twelve triaxial tests. (a), (c) and (e) Stress Paths in $p' - q$ diagram, (b), (d) and (f) $\varepsilon_1 - q - \varepsilon_v$ curves.

AGEP: Environmental Engineering papers

12 Lade, P.V., Ibsen, L.B. (1997). A study of the phase transformation and the characteristic lines of sand behaviour. Proc. Int. Symp. on Deformation and Progressive Failure in Geomechanics, Nagoya, Oct. 1997, pp. 353-359. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9702.

13 Bødker, L., Steenfelt, J.S. (1997). Vurdering af lodrette flytningsamplityder for maskinfundament, Color Print, Vadum (Evaluation of displacement amplitudes for printing machine foundation; in Danish). *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9706.

14 Ibsen, L.B., Steenfelt, J.S. (1997). Vurdering af lodrette flytningsamplityder for maskinfundament Løkkensvejens kraftvarmeværk (Evaluation of displacement amplitudes for gas turbine machine foundation; in Danish. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9707.

15 Steenfelt, J.S. (1997). National R&D Report : Denmark. *Seminar on Soil Mechanics and Foundation Engineering R&D*, Delft 13-14 February 1997. pp 4. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9708.

16 Lemonnier, P. and Soubra, A. H. (1997). Validation of the recent development of the displacement method - geogrid reinforced wall. *Colloquy EC97 on the comparison between experimental and numerical results*, Strasbourg, France. Vol.1, pp. 95-102. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9712.

17 Lemonnier, P. & Soubra, A. H. (1997). Recent development of the displacement method for the design of geosynthetically reinforced slopes - Comparative case study. *Colloquy on geosynthetics, Rencontres97, CFG*, Reims, France, Vol. 2, pp. 28AF-31AF (10pp). Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9713.

18 Lemonnier, P., Soubra, A. H. & Kastner, R. (1997). Variational displacement method for geosynthetically reinforced slope stability analysis : I. Local stability. Geotextiles and Geomembranes 16 (1998) pp 1-25. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9714.

19 Lemonnier, P., Soubra, A. H. & Kastner, R. (1997). Variational displacement method for geosynthetically reinforced slope stability analysis : II. Global stability. Geotextiles and Geomembranes 16 (1998) pp 27-44. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9715.

20 Ibsen, L.B. (1998). Analysis of Horizontal Bearing Capacity of Caisson Breakwater. 2nd PROVERS Workshop, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9802.

21 Ibsen, L.B. (1998). Advanced Numerical Analysis of Caisson Breakwater. *2nd PROVERS Workshop*, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9803.

22 Ibsen, L.B., Lade P.V. (1998). The Role of the Characteristic Line in Static Soil Behavior. *Proc. 4th International Workshop on Localization and Bifurcation Theory for Soil and Rocks.* Gifu, Japan. Balkema 1998. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9804.

AGEP: Environmental Engineering papers

23 Ibsen, L.B., Lade, P.V. (1998). The Strength and Deformation Characteristics of Sand Beneath Vertical Breakwaters Subjected to Wave Loading. 2nd PROVERS Workshop, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9805.

24 Steenfelt, J.S., Ibsen, L.B. (1998). The geodynamic approach - problem or possibility? Key Note Lecture, *Proc. Nordic Geotechnical Meeting, NGM-96, Reykjavik, V*ol 2, pp 14. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9809.

25 Lemonnier, P., Gotteland, Ph. and Soubra, A. H. (1998). Recent developments of the displacement method. *Proc. 6th Int. Conf. on Geosynthetics. Atlanta, USA*, Vol 2, pp 507-510. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9814.

26 Praastrup, U., Jakobsen, K.P., Ibsen, L.B. (1998). On the choice of strain measures in geo-mechanics. 12th Young Geotechnical Engineers Conference, Tallin, Estonia. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9815.

27 Ibsen, L.B. (1998). The mechanism controlling static liquefaction and cyclic strength of sand. Proc. Int. Workshop on Physics and Mechanics of Soil Liquefaction, Baltimore. A.A.Balkema, ISBN 9058090388, pp 29-39. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9816.

28 Ibsen, L.B., Jakobsen, K.P. (1998). Limit State Equations for Stability and Deformation. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9828.

29 Praastrup, U., Ibsen, L.B., Lade P.V. (1999). Presentation of Stress Points in the Customised Octahedral Plane. Published in *Proc. 13th ASCE Engineering Mechanics Division Conference,* June 13-16, 1999, Baltimore, USA, 6 pages. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9906.

30 Praastrup, U., Ibsen, L.B., Lade P.V. (1999). A Generic Stress Surface Introduced in the Customised Octahedral Plane. Proc. *7th Int. Symp. on Numerical Models in Geomechanics,* Graz, Austria, Sept. 1.-3. 1999. Balkema, Rotterdam ISBN 90 5809 095 7, pp 71-76. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9907.

31 Ibsen, L.B., Lade P.V. (1999). Effects of Nonuniform Stresses and Strains on Measured Characteristic States. Proc. *2nd Int. Symp. Pre-failure Deformation Characteristics of Geomaterials*, IS Torino 99, Sept. 26.-29.1999, pp 897-904. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9908.

32 Jakobsen, K.P., Praastrup, U., Ibsen, L.B. (1999). The influence of the stress path on the characteristic stress state. Proc. *2nd Int. Symp. Pre-failure Deformation Characteristics of Geomaterials*, IS Torino 99, Sept. 26.-29.1999, pp 659-666. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9909.

33 Praastrup, U., Jakobsen, K.P., Ibsen, L.B. (1999). Two Theoretically Consistent Methods for Analysing Triaxial Tests. Published in *Computers and Geotechnics,* Vol. 25(1999), pp. 157-170. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9912.

34 Ibsen, L.B. (1999). Cyclic Fatigue Model. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9916.